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# FREE CONVECTION NANOFLUID FLOW OVER A STRETCHING PLATE EMBEDDED IN MHD POROUS MEDIUM: A NUMERICAL INVESTIGATION

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***ABSTRACT***

***A numerical investigation of unsteady free convection MHD nanofluid flow and heat transfer over a stretching vertical flat plate embedded in a porous medium is done. Magnetic field of constant strength is applied in transverse direction. With the help of similarity variables, the governing equations are solved. Skin-friction coefficient and Nusselt number are calculated in non-dimensional form. The effect of various physical parameters including in governing equations are shown by graphs.***

***KEYWORDS: Nanofluid, MHD, Unsteady, Grash of Number, Stretching Sheet, Nusselt Number, Skin Friction Coefficient & Runge-Kutta Method***

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**Original Article**

## NOMENCLATURE

| | | | |
|---|---|---|---|
| $(\rho C_P)_f$ | heat capacity of fluid | $(\rho C_P)_s$ | heat capacity of nanoparticles |
| $\nu_f$ | kinematic viscosity of fluid | $(\rho C_P)_{nf}$ | heat capacity of nanofluid |
| $\mu_{nf}$ | Nanofluid viscosity | $\mu_f$ | Dynamic viscosity of base fluid |
| $\rho_{nf}$ | Density of nano-particles | $T_\infty$ | Free stream temperature |
| $U_w$ | stretching velocity | M | Magnetic number |
| Gr | Grash of number | Nu | Nusselt number |
| $B_0$ | Applied megnetic field | Pr | Prandtl number |
| S | Unsteady parameter | u | velocity component along x direction |
| K | Porosity of fluid | v | velocity component along y direction |
| T | Temperature | $\eta$ | similarity variable |
| $\phi$ | Volume fraction of solid particles | $\theta$ | Non dimensional tempareture |
| $\beta$ | Coefficient of thermal expansion | $\rho_f$ | Density of fluid |
| $\kappa_{nf}$ | Thermal conductivity of nanoparticles | $\rho_s$ | Density of solid |
| $\kappa_s$ | Thermal conductivity of base fluid | $\rho_f$ | Density of fluid |
| $\kappa_f$ | thermal conductivity of fluid | | |

## 1. INTRODUCTION

Now-a-days in industries, auto mobiles and thermal reactors water, ethylene, oil and glycol are used as convectional fluids. Thermal conductivity of these fluids is low which cause high manufacturing cost and maintenance. In heat transfer area traditional fluids are replaced by nanofluid because of their brilliant characteristics of enhanced/reduced heat. The application area of MHD nanofluid free convection flow and heat transfer of nanofluid through vertical stretching sheet covers industrial, scientific and engineering researches. The new field of application of fluid through stretching sheet is geothermal energy. Smaller size radiators and high performance can be maintained using nanofluid as coolants. Heat transfer rate can be improved by using MHD fluids. Extrusion, plasma studies, hot rolling, wire drawing, filaments, polymer sheet production, spinning, melt and production of rubber and plastic items are some application field of nanofluid on vertical plate according to Shankar and Yirga

(2013). During solution of viscous fluid over plate imposing stretching and velocity which is not fixed, Crane (1970) introduced the concept of stretching surface. Gorla and Zinalabedini (1987) investigated non-similar free convection mode of energy transfer on vertical plate applying variable temperature conditions. A numerical solution of energy process in rectangular enclosures which are heated partially and filled with nanofluid is solved by Oztop and Nada (2008). In a series of numerical solution of laminar flow, Khan and Pop (2010) contributed for flat sheet. Regarding thermal radiation effects mixed convection flow and heat transfer over sheet embedded in porous was explored by Mukhapadhyay (2009). Fang et al. (2010) examined MHD viscous nanofluid flow over a sheet when permeable shirking is applied. The closed form solution of nanofluid flow past a stretching sheet is solved by Hassani et al. (2011) in context of boundary layer. Makinde and Aziz (2011) worked out on the problem of nanofluid past a stretching sheet by applying convective boundary conditions. By assuming transverse magnetic field, the solution of natural convection nanofluid flow and heat transfer has been found out by Hamad (2011) in analytic form. Chamkha et al. (2011) addressed the effect of heat generation and melting on stretching sheet in horizontal direction for unsteady nanofluid. Khan and Pop (2011) solving a similarity form solution of nanofluid flow for horizontal plate saturated in porous medium regarding boundary. In this series, Ferdows et al. (2012) worked out on MHD mixed convective flow over surface under exponentially stretching. The effect of viscous chemical reaction and dissipation on governing field of heat and mass over stretching surface in account of MHD nanofluid are visualized graphically by Yohannes and Shankar (2013). Vendabie and Sarojamma (2014) elaborated the nature of flow through stretching surface under consideration of magnetic field and heat generation assuming flow to be unsteady nanofluid. The numerical solution of nanofluid flow and heat transfer over exponentially stretching sheet for 3-D taking $H_2O$ as base fluid is investigated by Nadeem et al. (2014). Yirga and Tesfay (2015) analyzed graphically the effect of various parameters including magnetic field parameter and Soret effects on nanofluid mass and heat transfer on permeable stretching sheet. Mahatha et al. (2015) worked out for incompressible and viscous over a stretching surface for nanofluid. They have taken MHD as electrically conducting fluid assuming partially slip velocity, nonlinear thermal radiation and Newtonian heating. In view of convective heating boundary condition, Ibrahim and Haq (2016) studied nanofluid flow of surface assuming stagnation point.

In this paper, MHD nanofluid flow over a vertical plate in context of unsteady free convection embedded in porous medium under consideration of variable temperature is investigated. A magnetic field of constant strength is applied in transverse direction. The governing equations are solved in the form of stream function using similarity variables. The heat transfer coefficient and skin-friction coefficient is calculated in tabular form. The effect of various parameters included in governing field is shown through graphs.

## 2. THE PHYSICAL DESCRIPTION OF PROBLEM

The study for flow and energy transfer through a plate saturated in a porous medium has been found out under consideration of stretching and unsteadily in nature. The x-axis is taken in vertical direction along plate and y-axis normal to the plate. Stretching velocity $U_w=\dfrac{bx}{1-at}$ is used and temperature is defined as $T_w=\dfrac{cx}{1-at}$. The plate is of free length vertical direction, so $\dfrac{\partial}{\partial x}(.)=0$.

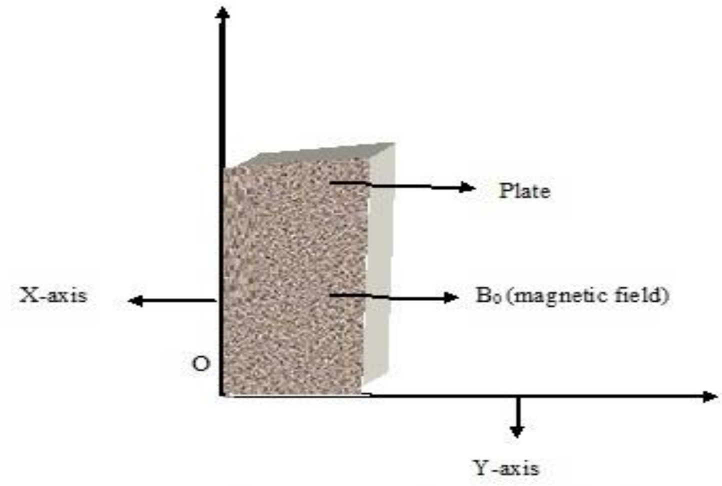


**Figure 1: Physical Diagram of the Problem.**

**2.1 Physical diagram of the Problem**

Applying Bossinesq's approximation and prescribed boundary conditions, the governing equations are converted as

The continuity equation

$$\frac{\partial v}{\partial y}=0 \tag{1}$$

N-S equation of momentum

$$\frac{\partial u}{\partial t}+v\frac{\partial u}{\partial y}=\frac{1}{\rho_{nf}}\left[\mu_{nf}\frac{\partial^2 u}{\partial y^2}+g(\rho\beta)_{nf}(T-T_\infty)-\mu_{nf}\frac{u}{K}-\sigma B_0^2 u\right] \tag{2}$$

The governing equation of temperature field is

$$\frac{\partial T}{\partial t}+v\frac{\partial T}{\partial y}=\frac{\kappa_{nf}}{(\rho C_p)_{nf}}\frac{\partial^2 T}{\partial y^2} \tag{3}$$

The imposed boundary conditions are given as:

$$\left.\begin{array}{l} y=0: \quad v=0,\ u=U_w=\dfrac{bx}{1-at}\,(\text{Stretching velocity}) \\ T=T_w=\dfrac{cx}{1-at} \\ y\to\infty: \quad v\to 0,\ u\to 0,\ T\to T_\infty \end{array}\right\} \tag{4}$$

## 3. MATHEMATICAL DISCRIPTION OF THE PROBLEM

Temperature and Stream function are used as given by Shankar and Yirga (2013) for changing governing equations in dimensionless form using stream function

$$\psi=\sqrt{\nu_f\, xU_w}\; f(\eta),\qquad \theta(\eta)=\frac{T-T_\infty}{T_w-T_\infty},\qquad \eta=\sqrt{\frac{U_w}{\nu_f\, x}}\;\Bigg\}\tag{5}$$

The various characteristics of nanofluid are used as defined by Oztop and Nada(2008)

$$\rho_{nf}=(1-\phi)\rho_f+\phi\rho_s\tag{6}$$

$$(C_p\rho)_{nf}=(1-\phi)(C_p\rho)_f+\phi(C_p\rho)_s\tag{7}$$

$$(\rho\beta)_{nf}=(1-\phi)(\rho\beta)_f+\phi(\rho\beta)_s\tag{8}$$

$$\mu_{nf}=\frac{\mu_f}{(1-\phi)^{2.5}}\ \text{(Maxwell-model)(9)}\tag{9}$$

The Maxwell-Garnetts model (1962) is used for the effective thermal conductivity of nanofluid defined as

$$\kappa_{nf}=\kappa_f\,\frac{\kappa_s+2\kappa_f-2(\kappa_f-\kappa_s)\phi}{\kappa_s+2\kappa_f+(\kappa_f-\kappa_s)\phi}\tag{10}$$

Inserting the similarity transformation defined by (5) and nanofluid physical properties as given in equations (6)-(10) into the equation (2), energy equation (3) and corresponding conditions given by (4). The dimensionless form of governing equations are converted respectively as

$$f'''=\frac{1}{AB}\left[S\frac{\eta}{2}f''-ff''+\left(S+ABP+M^2\right)f'-CAGr\theta\right]\tag{11}$$

$$S\frac{\eta}{2}\theta'-f\theta'=\frac{ED}{\Pr}\theta''\tag{12}$$

The dimensionless forms of boundary conditions are:

$$\left.\begin{array}{ll}\eta=0: & f(\eta)=0,\ \ f'(\eta)=1,\ \ \theta(\eta)=1\\ \eta\to\infty: & f\to 0,\ \ f'\to 0,\ \ \theta\to 0\end{array}\right\}\tag{13}$$

Included physical parameters are listed below.

$$S=\frac{a}{b}\text{(unsteady parameter)},\qquad M^2=\frac{\sigma B_0^2}{\rho_f U_w}x\ \text{(magnetic number)}$$

$$Gr=\beta_f\left(T_w-T_\infty\right)\frac{x}{U_w}\text{(Grashof number)},\quad P=\frac{\nu_f}{KU_w}x\ (\text{porous medium par}ameter)$$

$$\frac{1}{A}=(1-\varphi)+\varphi\frac{\rho_s}{\rho_f},\qquad B=\frac{1}{(1-\varphi)^{2.5}},\qquad C=(1-\varphi)+\varphi\frac{(\rho\beta)_s}{(\rho\beta)_f}$$

$$\frac{1}{D}=(1-\varphi)+\varphi\frac{(\rho cp)_s}{(\rho cp)_f},\qquad E=\frac{K_s+2K_f-2\varphi(K_f-K_s)}{K_s+2K_f+\varphi(K_f-K_s)}$$

## 4. SOLUTION DESCRIPTIONS

Shooting method is used for solving governing equations (11) and (12). In this method, unknown boundary conditions are identified first and assumed values are considered. In this way, the conditions prescribed on boundary are transferred to conditions on initial value. Then R-K method is used to improve unknown initial boundaries and improved continuously. Final boundary conditions are approached by solving IVP over the entire domain.

### 4.1 Solution for Velocity Components

Both the velocity components are defined as

$$\left.\begin{aligned} u&=\frac{\partial\psi}{\partial y}=U_w\, f' \\ v&=-\frac{\partial\psi}{\partial x}=-\sqrt{\frac{b\nu_f}{(1-at)}}\, f \end{aligned}\right\} \tag{14}$$

### 4.2 Heat Transfer Coefficient

The local heat transfer coefficient (Nu) in non-dimensional form is given as

$$Nu_x=\frac{xq_w}{K_f(T_w-T_\infty)},$$

$$\therefore Nu_x=-E\frac{1}{\sqrt{\mathrm{Re}_x}}\theta'(0) \tag{15}$$

### 4.3 Skin Friction Coefficient

Skin friction is given as

$$C_f=\left(\frac{\partial u}{\partial y}\right)_{y=0} \tag{16}$$

The non-dimensional form of coefficient of skin friction

$$C_f=-2AB\frac{1}{\sqrt{\mathrm{Re}_x}}f''(0) \tag{17}$$

## 5. RESULTS AND DISCUSSIONS

The graphs for velocity and temperature field over a stretching sheet are drawn for $H_2O$-Cu nanofluid. The effect of

parameters introduced in momentum and energy equations are depicted by graphs. One variable is changed and remaining are fixed and given specified values.

### 5.1 Effects on Velocity Profiles

It is presented by figure (2) that there is a hindrance to fluid flow when jump in solid volume fraction of nanoparticles occur that means $f'(\eta)$ is decreased. Figure (3) portrays the effect of Grash of number (Gr) on velocity profile which is growing due to high temperature. It can be observed that behavior of fluid velocity is reciprocal to the porous medium parameter P as depicted by figure (4). As the strength of magnetic force is increased, it causes loss in flow transfer due to opposite direction force named as Lorentzian force and same is conveyed in figure (5). The effects are more dominated for powerful magnetic force. The effect of unsteady parameter on velocity distribution $f'(\eta)$ are shown by figure (6).

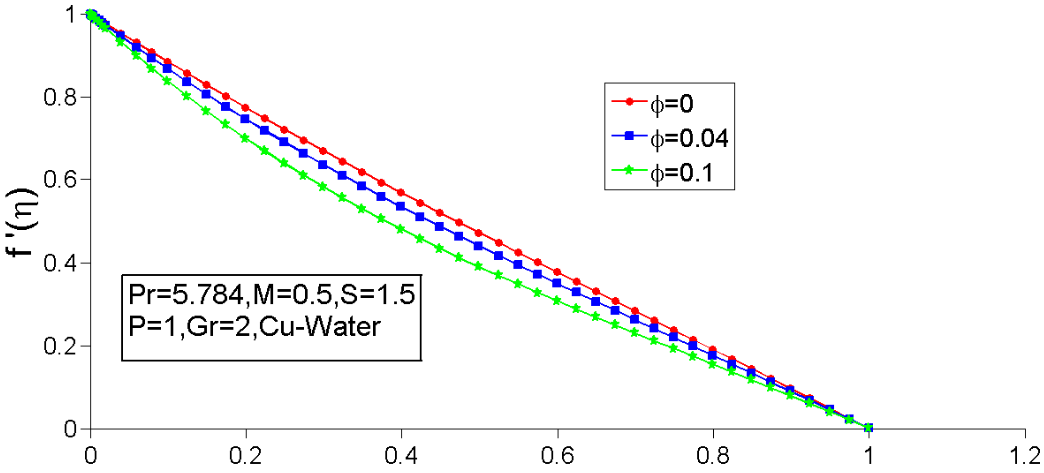


**Figure 2: Effect of $\phi$ On Velocity $f'(\eta)$**

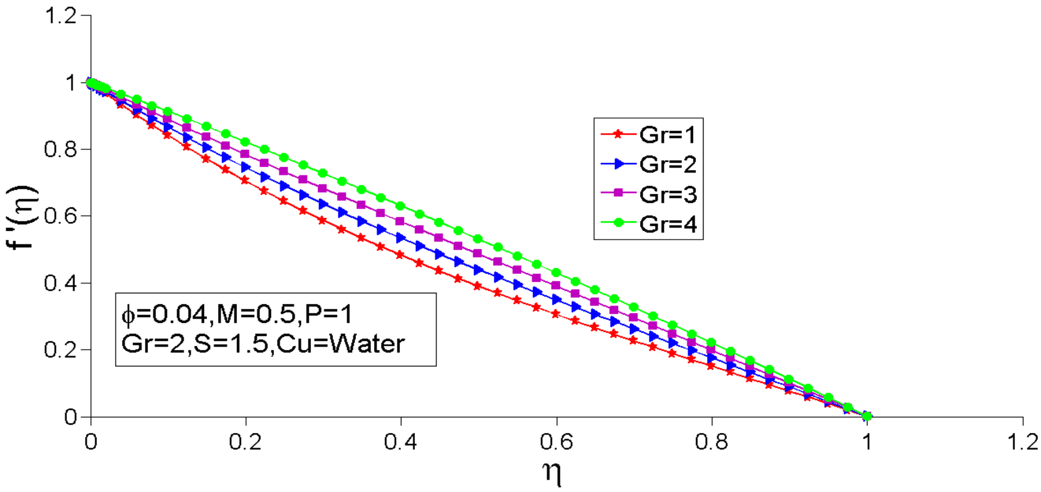


**Figure 3: Effect of Gr on Velocity $f'(\eta)$**

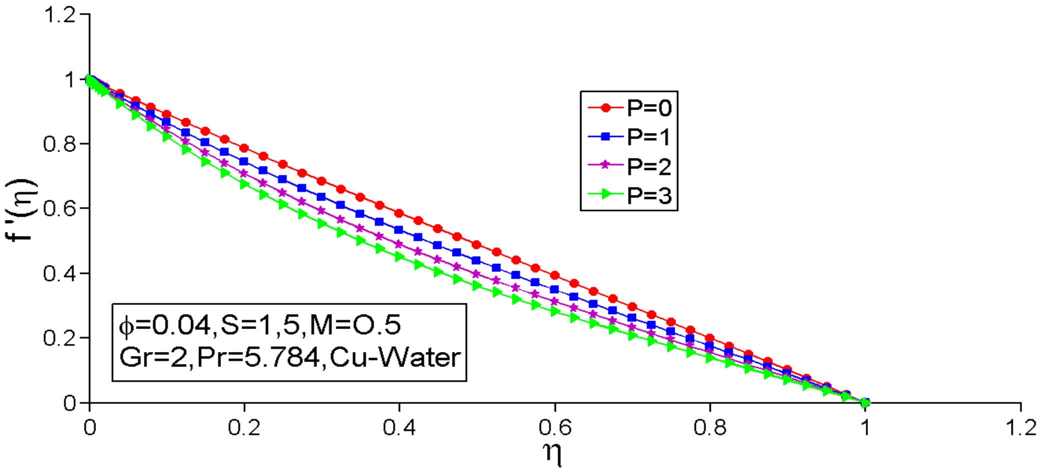


**Figure 4: Effect of P on Velocity** $f'(\eta)$

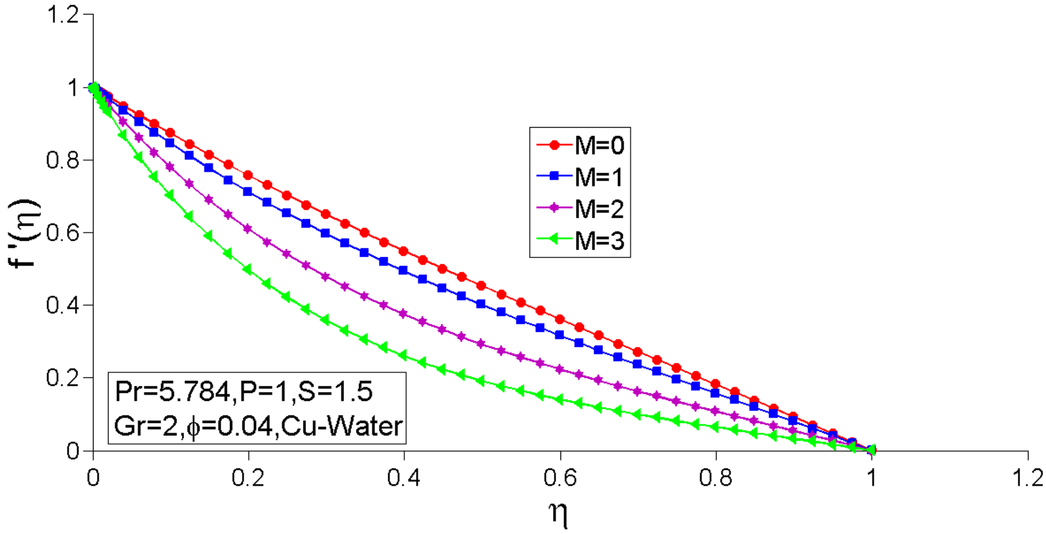


**Figure 5: Effect of M on Velocity** $f'(\eta)$

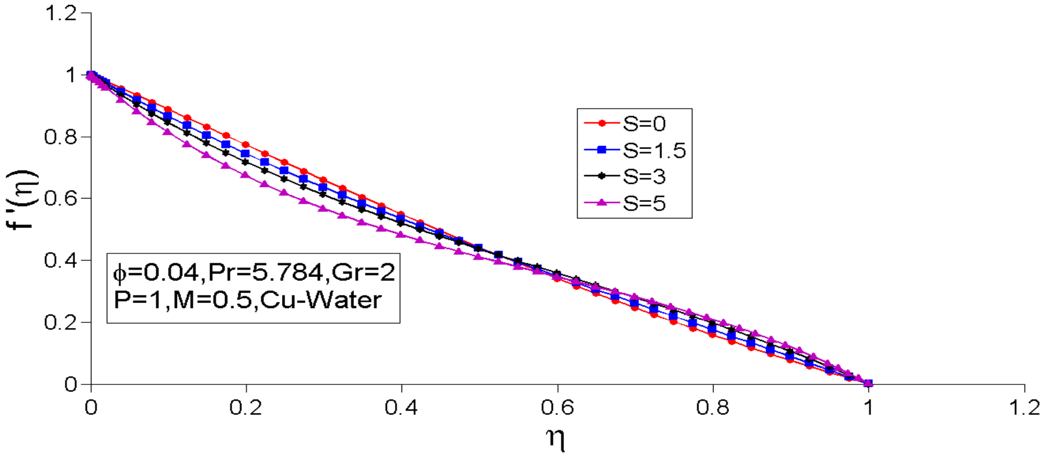


**Figure 6: Effect of S on Velocity** $f'(\eta)$

### 5.2 Effects on Temperature Profiles

The figure (7) shows enhance heat characteristic of nanofluid which state that temperature distribution $\theta(\eta)$ will be increased for $\phi$. The figure (8) conveys that higher value of Gr reduces heat transfer. The trend of M and P is same for temperature distribution as demonstrated in the figure (9)-(10) which is same as of Poornima and Reddy (2013). It is presented through the figure (11) that more energy is supplied for higher value of S. It can be seen by the figure that heat transfer is minimum when unsteady parameter is vanished. The increased value of Pr causes less thermal diffusivity due to which energy is increased as in figure (12).

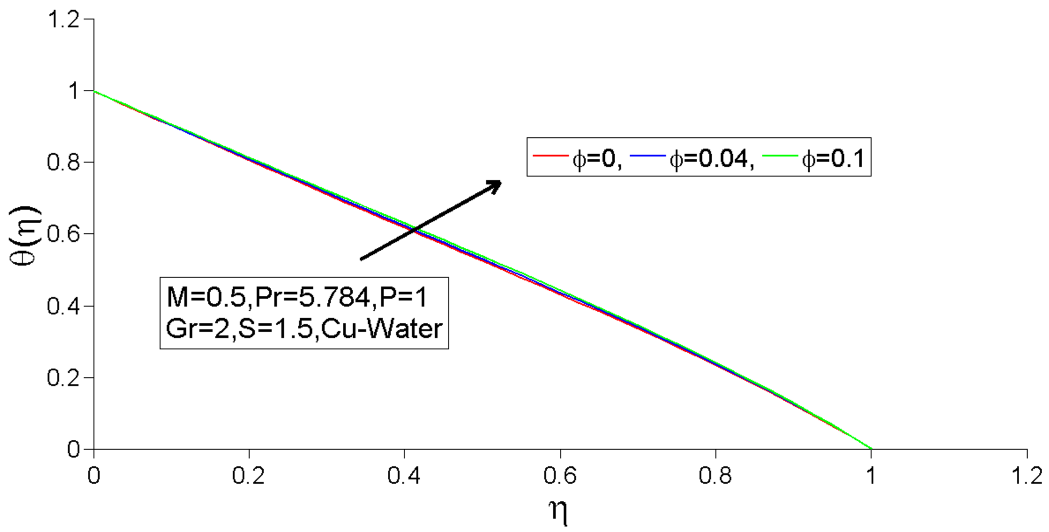


**Figure 7: Effect of $\phi$ on Temperature $\theta(\eta)$**

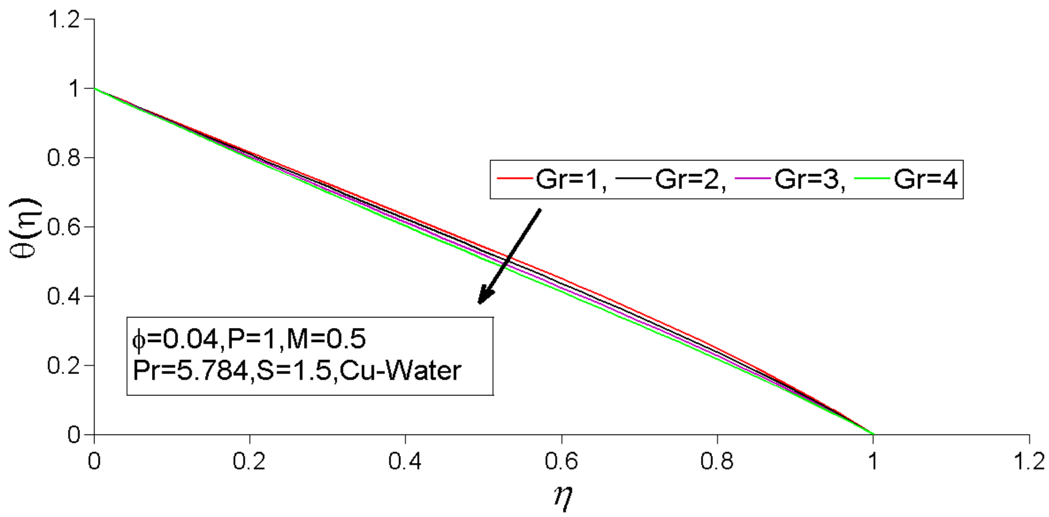


**Figure 8: Effect of Gr on Temperature $\theta(\eta)$**

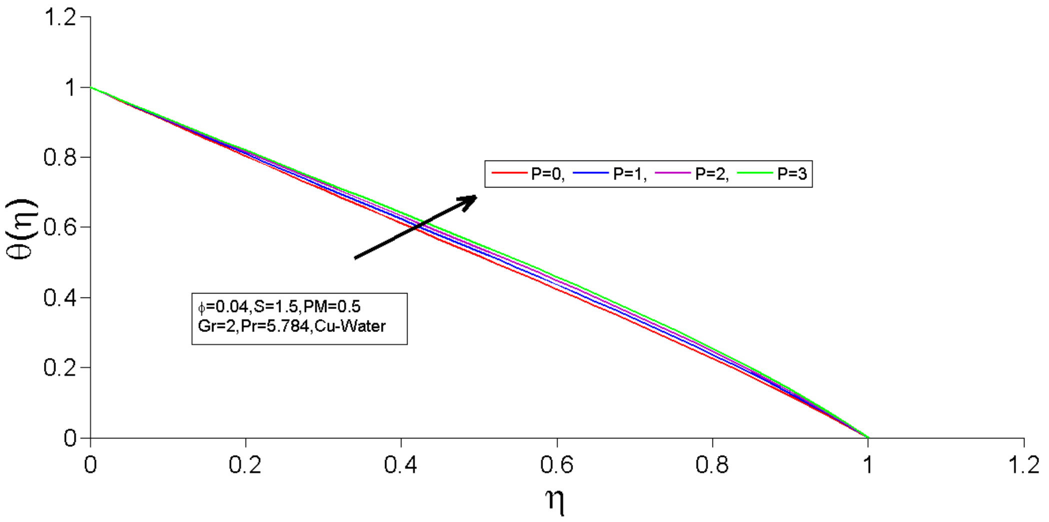


**Figure 9: Effect of P on Temperature** $\theta(\eta)$ **.**

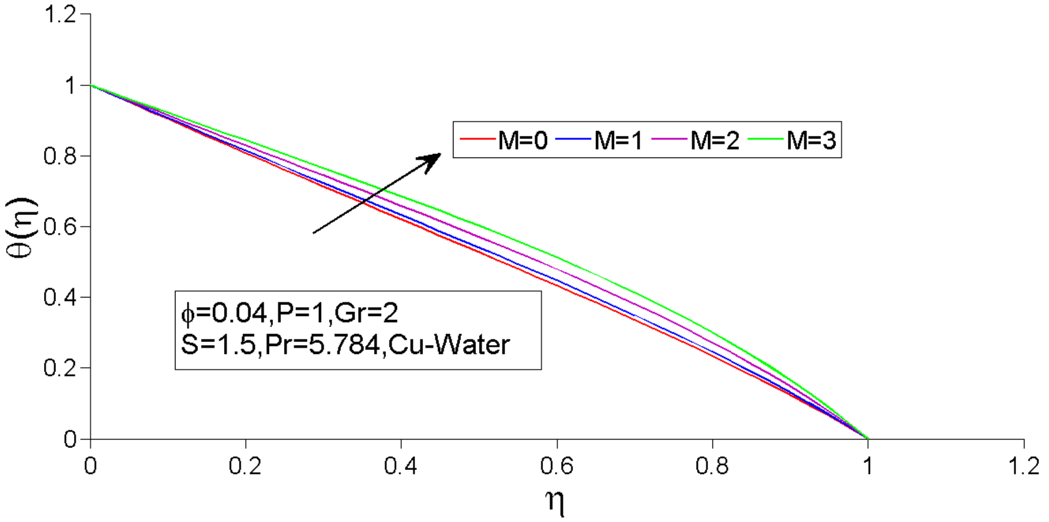


**Figure 10: Effect of M on Temperature** $\theta(\eta)$

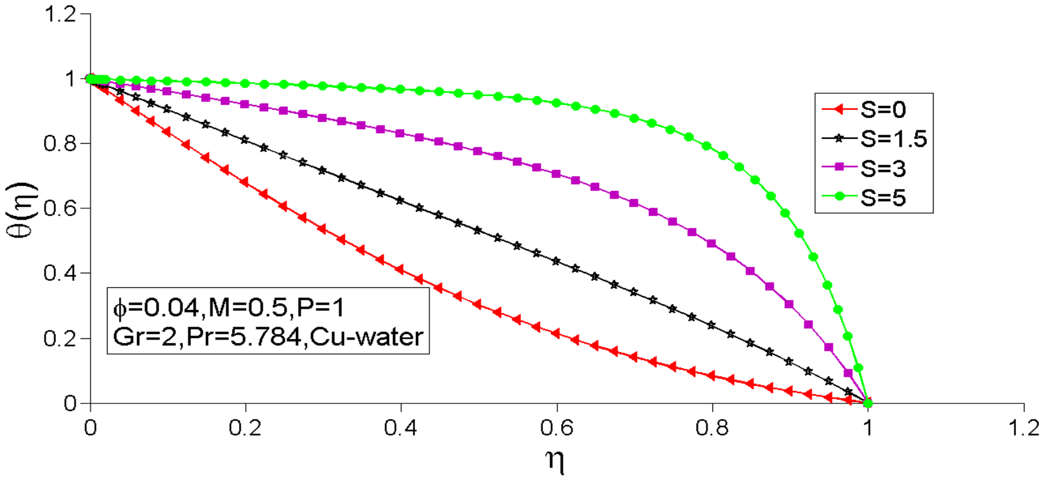


**Figure 11: Effect of S on Temperature** $\theta(\eta)$

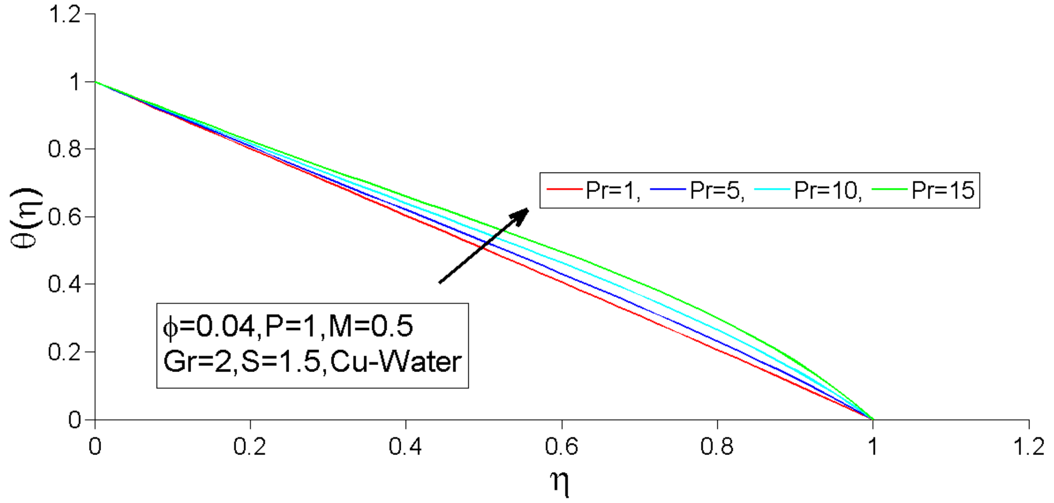


**Figure 12: Effect of Pr on Temperature $\theta(\eta)$**

### 5.3 Effects on Skin Friction Coefficient

The effect of different physical parameters including P, $\phi$, Gr, M and S for various nanofluids termed as $Al_2O_3$- $H_2O$,Cu-$H_2O$,Ag-$H_2O$ on skin friction coefficient are tabulated in Table 1. Drag force increases with increment in magnetic number because of Lorentz drag force. From table 1, it can be observed that $H_2O$-Ag and $H_2O$-$Al_2O_3$ nanofluid has highest and lowest skin friction coefficient respectively. So $H_2O$-Ag nanofluid has high skin friction comparatively to water Aluminia nanofluid. Table 1 also presents that there is proportional relation between $C_f$, $(\phi)$, Pr, Gr, S and P. It is distinguished that $H_2O$-$Al_2O_3$ has highest skin-friction in respect of all included parameters.

### 5.4 Effects on Heat Transfer Coefficient

The effect of different physical parameters including P, $\phi$, Gr, M and S for various nanofluids termed as $Al_2O_3$- $H_2O$,Cu-$H_2O$,Ag-$H_2O$ on Nusselt Number are tabulated in Table 2. It is reported from the table 2 that there is reciprocal relation between Nussult number porosity parameter (P) and magnetic number (M). The increased value of Pr causes less thermal diffusivity due to which energy is increased or Nusselt number has higher value. Nusselt number also has decreasing trend in increasing direction of S for nanofluids $H_2O$-$Al_2O_3$, $H_2O$-Cu and $H_2O$-Ag. The behavior of Gr and Nu have same trend. Due to high thermal capacity of silver, $H_2O$-Ag has highest heat transfer rate.

## 6. CONCLUSIONS

In the presence of porous medium momentum and energy transfer through unsteadily plate imposing stretching and unsteadily nature theoretical and graphical analysis has been done. The below mentioned remarkable conclusions are drawn:

- Fluid flow is of decreasing nature with increment in value of P, M and $\phi$.
- Fluid flow and Gr has same nature.
- Fluid flow is increasing for S upto $\eta$=0.55(approx.) and for higher value has reverse effect.

- Temperature flow and P, M, Pr, S and $\phi$ has same behavior.
- Gr has reverse effect on heat transfer rate.
- There is proportional relation between Drag force and M.
- Nu decreases with increment in amount of M, P, Pr, S and $\phi$.

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## APPENDIEX

**Table 1: Effect of Physical Parameters on Coefficient of Skin-Friction ($C_f$) of Different Type of Nanofluids**

| Physical Parameters | | | | | | Nanofluid | | |
|---|---|---|---|---|---|---|---|---|
| | | | | | | $H_2O$-$Al_2O_3$ | $H_2O$-Cu | $H_2O$-Ag |
| M | Pr | P | $\phi$ | Gr | S | $C_f$ | $C_f$ | $C_f$ |
| 0 | 5.784 | 1 | 0.04 | 2 | 1.5 | 1.253158 | 1.32489 | 1.345468 |
| 0.5 | 5.784 | 1 | 0.04 | 2 | 1.5 | 1.332998 | 1.416227 | 1.440311 |
| 1 | 5.784 | 1 | 0.04 | 2 | 1.5 | 1.560647 | 1.675028 | 1.708553 |
| 2 | 5.784 | 1 | 0.04 | 2 | 1.5 | 2.335491 | 2.541652 | 2.602642 |
| 0.5 | 0.7 | 1 | 0.04 | 2 | 1.5 | 1.343393 | 1.42776 | 1.452273 |
| 0.5 | 1 | 1 | 0.04 | 2 | 1.5 | 1.342828 | 1.427091 | 1.451623 |
| 0.5 | 1.5 | 1 | 0.04 | 2 | 1.5 | 1.341817 | 1.425973 | 1.450458 |
| 0.5 | 2 | 1 | 0.04 | 2 | 1.5 | 1.340801 | 1.42485 | 1.449289 |
| 0.5 | 5.784 | 1 | 0.04 | 2 | 1.5 | 1.332998 | 1.416227 | 1.440311 |
| 0.5 | 5.784 | 0 | 0.04 | 2 | 1.5 | 1.003724 | 1.09775 | 1.125056 |
| 0.5 | 5.784 | 1 | 0.04 | 2 | 1.5 | 1.332998 | 1.416227 | 1.440311 |
| 0.5 | 5.784 | 2 | 0.04 | 2 | 1.5 | 1.62991 | 1.704693 | 1.726253 |
| 0.5 | 5.784 | 3 | 0.04 | 2 | 1.5 | 1.900896 | 1.968923 | 1.988464 |
| 0.5 | 5.784 | 1 | 0.04 | 2 | 1.5 | 1.332998 | 1.416227 | 1.440311 |
| 0.5 | 5.784 | 1 | 0.1 | 2 | 1.5 | 1.574072 | 1.790115 | 1.850355 |
| 0.5 | 5.784 | 1 | 0.04 | 1 | 1.5 | 1.628536 | 1.713603 | 1.739026 |
| 0.5 | 5.784 | 1 | 0.04 | 2 | 1.5 | 1.332998 | 1.416227 | 1.440311 |
| 0.5 | 5.784 | 1 | 0.04 | 3 | 1.5 | 1.043372 | 1.124684 | 1.147499 |
| 0.5 | 5.784 | 1 | 0.04 | 4 | 1.5 | 0.759304 | 0.838629 | 0.860241 |
| 0.5 | 5.784 | 1 | 0.04 | 2 | 0 | 1.069526 | 1.094935 | 1.102844 |
| 0.5 | 5.784 | 1 | 0.04 | 2 | 1 | 1.248805 | 1.313378 | 1.33232 |
| 0.5 | 5.784 | 1 | 0.04 | 2 | 1.5 | 1.332998 | 1.416227 | 1.440311 |
| 0.5 | 5.784 | 1 | 0.04 | 2 | 3 | 1.589226 | 1.724743 | 1.763609 |
| 0.5 | 5.784 | 1 | 0.04 | 2 | 5 | 1.974349 | 2.166915 | 2.223248 |

**Table 2: Effect of Physical Parameters on Heat Transfer Coefficient (Nusselt Number) $Nu_x$ of Different Type of Nanofluids**

| Physical Parameters | | | | | | Nanofluid | | |
|---|---|---|---|---|---|---|---|---|
| | | | | | | $H_2O$-$Al_2O_3$ | $H_2O$-Cu | $H_2O$-Ag |
| M | Pr | P | $\phi$ | Gr | S | Nu | Nu | Nu |
| 0 | 5.784 | 1 | 0.04 | 2 | 1.5 | 0.973095 | 0.966409 | 0.964098 |
| 0.5 | 5.784 | 1 | 0.04 | 2 | 1.5 | 0.965023 | 0.957399 | 0.954696 |
| 1 | 5.784 | 1 | 0.04 | 2 | 1.5 | 0.942609 | 0.932643 | 0.928947 |
| 2 | 5.784 | 1 | 0.04 | 2 | 1.5 | 0.873271 | 0.85837 | 0.852401 |
| 0.5 | 0.7 | 1 | 0.04 | 2 | 1.5 | 0.995795 | 0.994997 | 0.994522 |
| 0.5 | 1 | 1 | 0.04 | 2 | 1.5 | 0.994148 | 0.992839 | 0.992378 |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 0.5 | 1.5 | 1 | 0.04 | 2 | 1.5 | 0.991189 | 0.989225 | 0.988532 |
| 0.5 | 2 | 1 | 0.04 | 2 | 1.5 | 0.988209 | 0.985588 | 0.984663 |
| 0.5 | 5.784 | 1 | 0.04 | 2 | 1.5 | 0.965023 | 0.957399 | 0.954696 |
| 0.5 | 5.784 | 1 | 0.04 | 2 | 1.5 | 0.965023 | 0.957399 | 0.954696 |
| 0.5 | 5.784 | 2 | 0.04 | 2 | 1.5 | 0.935969 | 0.92988 | 0.927293 |
| 0.5 | 5.784 | 3 | 0.04 | 2 | 1.5 | 0.910819 | 0.905952 | 0.903435 |
| 0.5 | 5.784 | 1 | 0 | 2 | 1.5 | 0.975324 | 0.975324 | 0.975324 |
| 0.5 | 5.784 | 1 | 0.04 | 2 | 1.5 | 0.965023 | 0.957399 | 0.954696 |
| 0.5 | 5.784 | 1 | 0.1 | 2 | 1.5 | 0.950415 | 0.935306 | 0.929274 |
| 0.5 | 5.784 | 1 | 0.04 | 1 | 1.5 | 0.933962 | 0.926684 | 0.923621 |
| 0.5 | 5.784 | 1 | 0.04 | 2 | 1.5 | 0.965023 | 0.957399 | 0.954696 |
| 0.5 | 5.784 | 1 | 0.04 | 3 | 1.5 | 0.995133 | 0.987186 | 0.984825 |
| 0.5 | 5.784 | 1 | 0.04 | 4 | 1.5 | 1.024324 | 1.016074 | 1.014037 |
| 0.5 | 5.784 | 1 | 0.04 | 2 | 0 | 1.667883 | 1.658433 | 1.66351 |
| 0.5 | 5.784 | 1 | 0.04 | 2 | 1 | 1.198691 | 1.190114 | 1.189972 |
| 0.5 | 5.784 | 1 | 0.04 | 2 | 1.5 | 0.965023 | 0.957399 | 0.954696 |
| 0.5 | 5.784 | 1 | 0.04 | 2 | 3 | 0.387218 | 0.383754 | 0.377315 |
| 0.5 | 5.784 | 1 | 0.04 | 2 | 5 | 0.063236 | 0.06302 | 0.060218 |